

Improved Approximation Algorithms for the Non-preemptive Speed-scaling Problem*

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Abstract

We are given a set of jobs, each one specified by its release date, its deadline and its processing volume (work), and a single (or a set of) speed-scalable processor(s). We adopt the standard model in speed-scaling in which if a processor runs at speed s then the energy consumption is s^α per time unit, where $\alpha > 1$. Our goal is to find a schedule respecting the release dates and the deadlines of the jobs so that the total energy consumption is minimized. While most previous works have studied the preemptive case of the problem, where a job may be interrupted and resumed later, we focus on the non-preemptive case where once a job starts its execution, it has to continue until its completion without any interruption. We propose improved approximation algorithms for the multiprocessor non-preemptive speed-scaling problem for particular families of instances, namely instances where all jobs have a common release date (or a common deadline), instances where all jobs are active at some time, and agreeable instances.

1 Introduction

One of the main mechanisms used for minimizing the energy consumption in computing systems and portable devices is the so called speed-scaling mechanism. In this setting, the speed of a processor may change dynamically. If the speed of the processor is $s(t)$ at time t then its power is $s(t)^\alpha$, for some constant $\alpha > 1$, and the energy consumption is the power integrated over time.

There is a series of papers in the speed-scaling literature. In their seminal paper, Yao et al. [14] proposed a polynomial-time algorithm for the energy minimization problem of scheduling a set of n jobs \mathcal{J} , where each job $J_j \in \mathcal{J}$ is characterized by its processing volume (work) w_j , its release date r_j and its deadline d_j , on a single speed-scalable processor, assuming that the preemption of the jobs, i.e. the possibility to interrupt the execution of a job and resume it later, is allowed. Most of the subsequent works on energy minimization in the speed-scaling setting allow jobs to preempt. However in practice, preemption causes an important overhead and it is sometimes even impossible, i.e., in the case where external resources are used. Hence, it is natural to disallow it. Only recently, Antoniadis and Huang, in [5], studied the non-preemptive version of the energy minimization problem introduced by Yao et al. [14], and they proved that the problem becomes strongly \mathcal{NP} -hard. They have also proposed a constant factor approximation algorithm.

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The same paper also studied the problem for a particular family of instances, namely *laminar* instances. Here for any two jobs J_j and $J_{j'}$ with $r_j \leq r_{j'}$ it holds that either $d_j \geq d_{j'}$ or $d_j \leq r_{j'}$. In fact, such instances typically arise when recursive calls in a program create new jobs. Another interesting family of instances studied in the literature is the family of *agreeable* instances. In an agreeable instance, for any two jobs J_j and $J_{j'}$ with $r_j \leq r_{j'}$ it holds that $d_j \leq d_{j'}$, i.e. latter released jobs have latter deadlines. Such instances may arise in situations where the goal is to maintain a fair service guarantee for the waiting time of jobs. Two more special families of instances that we use in our study below are the *clique* instances and the *pure-laminar* instances. In a clique instance, for any two jobs J_j and $J_{j'}$ with $r_j \leq r_{j'}$ it holds that $d_j \geq r_{j'}$. In other words, in a clique instance there is a time, T , where all jobs are active; for example $T = \min\{d_j, J_j \in \mathcal{J}\}$ or $T = \max\{r_j, J_j \in \mathcal{J}\}$. In a pure-laminar instance, for any two jobs J_j and $J_{j'}$ with $r_j \leq r_{j'}$ it holds that $d_j \geq d_{j'}$. Note that the family of pure-laminar instances is a special case of both laminar and clique instances. Finally, two other interesting special cases of all the above families, studied by several works in scheduling, are those where all the jobs have either a common release date or a common deadline.

1.1 Related work

As mentioned above, for the preemptive single-processor case, Yao et al. [14] proposed an optimal algorithm for finding a feasible schedule with minimum energy consumption. Using an extension of the classical three-field notation, this problem can be denoted as $S1|r_j, d_j, pmtn|E$. The multiprocessor case, $S|r_j, d_j, pmtn|E$, where there are m available processors has been solved optimally in polynomial time when preemption and migration of jobs are allowed [2, 4, 8]. The migration assumption means that a job may be interrupted and resumed on the same processor or on another processor. However, the parallel execution of parts of the same job is not allowed.

Albers et al. [3] considered the multiprocessor problem $S|r_j, d_j, pmtn, no-mig|E$, where the preemption of the jobs is allowed, but not their migration. They first studied the problem where each job has unit work. They proved that the problem is polynomial time solvable for instances with agreeable deadlines. For general instances with unit-work jobs, they proved that the problem becomes strongly \mathcal{NP} -hard and they proposed an $(\alpha^\alpha 2^{4\alpha})$ -approximation algorithm. For the case where the jobs have arbitrary works, the problem was proved to be \mathcal{NP} -hard even for instances with common release dates and common deadlines. Albers et al. proposed a $2(2 - \frac{1}{m})^\alpha$ -approximation algorithm for instances with common release dates, or common deadlines, and an $(\alpha^\alpha 2^{4\alpha})$ -approximation algorithm for instances with agreeable deadlines. Greiner et al. [11] gave a generic reduction transforming an optimal schedule for the multiprocessor problem with migration, $S|r_j, d_j, pmtn|E$, to a B_α -approximate solution for the multiprocessor problem with preemptions but without migration, $S|r_j, d_j, pmtn, no-mig|E$, where B_α is the α -th Bell number. This result holds only when $m \geq \alpha$.

It has to be noticed here that for the family of agreeable instances, and hence for their special families of instances (instances with common release dates and/or common deadlines), the assumption of preemption and no migration is equivalent to the non-preemptive assumption that we consider throughout this paper. More specifically, for agreeable instances, any preemptive schedule can be transformed into a non-preemptive one of the same energy consumption, where the execution of each job $J_j \in \mathcal{J}$ starts after the completion of any other job which is released before J_j . The correctness of this transformation can be easily proved by induction to the order where the jobs are released. Hence, the results of [3] and [11] for agreeable deadlines hold for the non-preemptive case as well.

Finally, the most closely related work is the work of Antoniadis and Huang [5] who consid-

ered the energy minimization single-processor non-preemptive speed-scaling problem. They first proved that the problem is \mathcal{NP} -hard even for pure-laminar instances. They also presented a $2^{4\alpha-3}$ -approximation algorithm for laminar instances and a $2^{5\alpha-4}$ -approximation algorithm for general instances. Notice that the polynomial-time algorithm for finding an optimal preemptive schedule presented in [14] returns a non-preemptive schedule when the input instance is agreeable.

In Table 1, we summarize the most related results of the literature. Several other results concerning scheduling problems in the speed-scaling setting have been presented, involving the optimization of some QoS criterion under a budget of energy, or the optimization of a linear combination of the energy consumption and some QoS criterion (see for example [7, 9, 13]). The interested reader can find more details in the recent survey [1].

Problem	Complexity	Approximation ratio	
		$m < \alpha$	$m \geq \alpha$
$S1 r_j, d_j, pmtn E$	Polynomial [14]	–	
$S r_j = 0, d_j = d, pmtn E$	Polynomial [10]	–	
$S r_j, d_j, pmtn E$	Polynomial [2, 4, 8]	–	
$S agreeable, w_j = 1, pmtn, no-mig E^{(*)}$	Polynomial [3]	–	
$S r_j, d_j, w_j = 1, pmtn, no-mig E$	\mathcal{NP} -hard ($m \geq 2$) [3]	$\alpha^\alpha 2^{4\alpha}$ [3]	B_α [11]
$S r_j = 0, d_j = d, pmtn, no-mig E^{(*)}$	\mathcal{NP} -hard [3]	$PTAS$ [12, 3]	
$S r_j = 0, d_j, pmtn, no-mig E^{(*)}$	\mathcal{NP} -hard	$2(2 - \frac{1}{m})^\alpha$ [3]	$\min\{2(2 - \frac{1}{m})^\alpha, B_\alpha\}$ [3, 11]
$S r_j, d_j = d, pmtn, no-mig E^{(*)}$	\mathcal{NP} -hard	$2(2 - \frac{1}{m})^\alpha$ [3]	$\min\{2(2 - \frac{1}{m})^\alpha, B_\alpha\}$ [3, 11]
$S agreeable, pmtn, no-mig E^{(*)}$	\mathcal{NP} -hard	$\alpha^\alpha 2^{4\alpha}$ [3]	B_α [11]
$S r_j, d_j, pmtn, no-mig E$	\mathcal{NP} -hard	–	B_α [11]
$S1 agreeable E$	Polynomial [14, 5]	–	
$S1 laminar E$	\mathcal{NP} -hard [5]	$2^{4\alpha-3}$ [5]	
$S1 r_j, d_j E$	\mathcal{NP} -hard	$2^{5\alpha-4}$ [5]	

Table 1: Complexity and approximability results. $(*)$ The problem is equivalent with the corresponding non-preemptive problem.

1.2 Our contribution

In this paper, we explore the approximability for the non-preemptive speed-scaling problem on multiprocessors for special families of instances. More specifically, in Section 3 we consider the multiprocessor case where the jobs have either common release dates or common deadlines. Recall that for these problems algorithms of approximation ratio $2(2 - \frac{1}{m})^\alpha$ have been presented in [3], while these results have been improved in [11] to B_α , if $\alpha \leq m$ and $\alpha \leq 5$. We further improve the approximation ratio to $(2 - \frac{1}{m})^{\alpha-1}$, for any value of α and m . In Section 5, we consider the agreeable multiprocessor case and we present a $(4(2 - \frac{1}{m}))^{\alpha-1}$ -approximation algorithm. However, for $\alpha \leq m$, a B_α -approximation algorithm for the case has been presented in [11]. Hence, this result dominates our approximation factor for $\alpha \leq m$. Even if for practical applications the assumption made in [11] that $\alpha \leq m$ is justified, from a theoretical point of view it is worthwhile to investigate the approximability of the problem when $\alpha > m$. For the latter case, our result improves the ratio of $\alpha^\alpha 2^{4\alpha}$ given in [3].

A summary of our results compared with the previously known results is given in Table 2.

1.3 Notation and Preliminaries

A fundamental property of optimal schedules in the speed-scaling model, which is also true for the problems we study in this paper, is that any job $J_j \in \mathcal{J}$ runs at a constant speed s_j due to the

Problem	Previous result		Our result
	$m < \alpha$	$m \geq \alpha$	
$S r_j = 0, d_j E$	$2(2 - \frac{1}{m})^\alpha$ [3]	$\min\{2(2 - \frac{1}{m})^\alpha, B_\alpha\}$ [3, 11]	$(2 - \frac{1}{m})^{\alpha-1}$
$S r_j, d_j = d E$	$2(2 - \frac{1}{m})^\alpha$ [3]	$\min\{2(2 - \frac{1}{m})^\alpha, B_\alpha\}$ [3, 11]	$(2 - \frac{1}{m})^{\alpha-1}$
$S \text{clique} E$	–	–	$(2(2 - \frac{1}{m}))^{\alpha-1}$
$S \text{agreeable} E$	$\alpha^\alpha 2^{4\alpha}$ [3]	B_α [11]	$(4(2 - \frac{1}{m}))^{\alpha-1} < 2^{3\alpha-3}$

Table 2: Previous known approximation ratios vs. our approximation ratios.

convexity of the speed-to-power function. Given a schedule \mathcal{S} and a job $J_j \in \mathcal{J}$, we denote by $E(\mathcal{S}, J_j) = w_j s_j^{\alpha-1}$ the energy consumed by the execution of J_j in \mathcal{S} and by $E(\mathcal{S}) = \sum_{j=1}^n E(\mathcal{S}, J_j)$ the total energy consumed by \mathcal{S} . Given an instance \mathcal{I} and a job $J_j \in \mathcal{J}$, we denote by $w_j(\mathcal{I})$ the work, by $r_j(\mathcal{I})$ the release date and by $d_j(\mathcal{I})$ the deadline of J_j in \mathcal{I} . We denote by \mathcal{S}^* an optimal non-preemptive schedule for the input instance \mathcal{I} . For each job $J_j \in \mathcal{J}$, we call the interval $[r_j, d_j]$ the *active interval* of J_j .

The following proposition has been proved in [5] for $S1|r_j, d_j|E$ but holds also for the corresponding problem on parallel processors.

Proposition 1 [5] *Suppose that the schedules \mathcal{S} and \mathcal{S}' process job J_j with speed s and s' respectively. Assume that $s \leq \gamma s'$ for some $\gamma \geq 1$. Then $E(\mathcal{S}, j) \leq \gamma^{\alpha-1} E(\mathcal{S}', j)$.*

2 From Preemptive to Non-preemptive Scheduling

In this paper, we explore the idea of transforming an optimal preemptive schedule into a non-preemptive schedule, guaranteeing that the energy consumption of the latter one does not increase more than a factor ρ . Unfortunately, in the following proposition we show that for general instances the ratio between an optimal non-preemptive schedule to an optimal preemptive one can be very large.

Proposition 2 *The ratio of the energy consumption of an optimal non-preemptive schedule to the energy consumption of an optimal preemptive schedule for the same instance of the single-processor speed-scaling problem can be $O(n^{\alpha-1})$.*

Proof. Consider the instance consisting of a single processor, $n - 1$ unit-work jobs, J_1, J_2, \dots, J_{n-1} , and the job J_n of work n . Each job J_j , $1 \leq j \leq n - 1$, has release date $r_j = 2j - 1$ and deadline $d_j = 2j$, while $r_n = 0$ and $d_n = 2n - 1$ (see Figure 1).

The optimal preemptive schedule \mathcal{S}_{pr} for this instance assigns to all jobs a speed equal to one. Each job J_j , $1 \leq j \leq n - 1$, is executed during its whole active interval, while J_n is executed during the remaining n unit length intervals. The total energy consumption of this schedule is $E(\mathcal{S}_{pr}) = (n - 1) \cdot 1^\alpha + n \cdot 1^\alpha = 2n - 1$.

An optimal non-preemptive schedule \mathcal{S}_{npr} for this instance assigns a speed $\frac{n+2}{3}$ to jobs J_1, J_n and J_2 and schedules them non-preemptively in this order between time 1 and 4. Moreover, in \mathcal{S}_{npr} each job J_j , $3 \leq j \leq n - 1$, is assigned a speed equal to one and it is executed during its whole active interval. The total energy consumption of this schedule is $E(\mathcal{S}_{npr}) = 3 \cdot (\frac{n+2}{3})^\alpha + (n - 3) \cdot 1^\alpha$.

Therefore, we have $\frac{E(\mathcal{S}_{npr})}{E(\mathcal{S}_{pr})} = \frac{3 \cdot (\frac{n+2}{3})^\alpha + (n-3) \cdot 1^\alpha}{2n-1} = O(n^{\alpha-1})$. ■

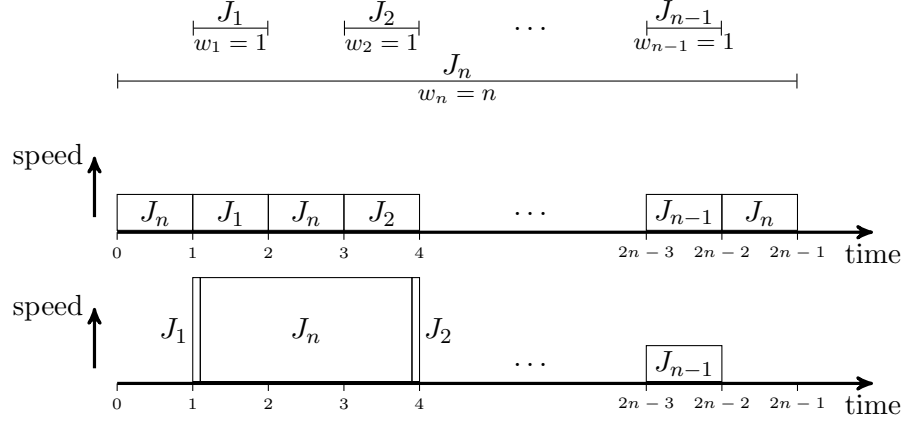


Figure 1: An instance of $S1|r_j, d_j|E$ for which the ratio of the energy consumption in an optimal non-preemptive schedule to the energy consumption in an optimal preemptive schedule is $O(n^{\alpha-1})$.

In what follows, we show that for some particular families of instances, for which the energy minimization multi-processor speed-scaling problem is known to be \mathcal{NP} -hard, it is possible to use the cost of an optimal preemptive schedule as a lower bound of the energy consumption of an optimal non-preemptive schedule in order to obtain good approximation ratios.

3 Common Release Dates or Common Deadlines

In this section we deal with $S|r_j = 0, d_j|E$ and $S|r_j, d_j = d|E$, which are \mathcal{NP} -hard as generalizations of $S|r_j = 0, d_j = d|E$. In fact, we will present an approximation algorithm for $S|r_j = 0, d_j|E$, which achieves a better ratio than the known approximation algorithms presented in [3] and [11] for any values of α and m . We also describe how to adapt this algorithm to an algorithm for $S|r_j, d_j = d|E$ of the same approximation ratio.

ALGORITHM CRD takes as input an instance \mathcal{I} of $S|r_j = 0, d_j|E$ and creates first an optimal preemptive schedule for \mathcal{I} , using one of the algorithms in [2, 4, 8]. The total execution time e_j of each job $J_j \in \mathcal{J}$ in this preemptive schedule is used to define an appropriate processing time p_j for J_j . Then, the algorithm schedules non-preemptively the jobs using these processing times according to the Earliest Deadline First policy, i.e., at every time that a machine becomes idle, the non-scheduled job with the minimum deadline is scheduled on it. The choice of the values of the p_j 's has been made in such a way that the algorithm completes all the jobs before their deadlines.

ALGORITHM CRD(\mathcal{I})

- 1: Create an optimal preemptive schedule \mathcal{S}_{pr} for \mathcal{I} ;
 - 2: Let e_j be the total execution time of the job $J_j \in \mathcal{J}$, in \mathcal{S}_{pr} ;
 - 3: Schedule the jobs with the Earliest Deadline First (EDF) policy, using the appropriate speed such that the processing time of the job $J_j \in \mathcal{J}$, is equal to $p_j = e_j / (2 - \frac{1}{m})$, obtaining the non-preemptive schedule \mathcal{S}_{npr} ;
 - 4: **return** \mathcal{S}_{npr} ;
-

Theorem 3 ALGORITHM CRD is a $(2 - \frac{1}{m})^{\alpha-1}$ -approximation algorithm for $S|r_j = 0, d_j|E$.

Proof. We first prove by induction that for the completion time C_j of the job $J_j \in \mathcal{J}$, it holds that $C_j \leq d_j$.

For each job J_j , $1 \leq j \leq m$, it holds that $C_j = p_j = e_j/(2 - \frac{1}{m}) \leq d_j$, since J_j is selected by the algorithm for $S|r_j, d_j, pmtn|E$ such that \mathcal{S}_{pr} to be feasible, and hence $e_j \leq d_j$. Assume that our hypothesis is true for all jobs in $\mathcal{J}_{k-1} = \{J_1, J_2, \dots, J_{k-1}\}$.

For the job J_k , it holds that $C_k \leq \frac{\sum_{j=1}^{k-1} p_j}{m} + p_k = (\frac{\sum_{j=1}^{k-1} e_j}{m} + e_k)/(2 - \frac{1}{m})$. As \mathcal{S}_{pr} is a feasible schedule and $d_k \geq d_j$ for each job $J_j \in \mathcal{J}_{k-1}$, it holds that $e_k \leq d_k$ and $\sum_{j=1}^k e_j \leq m \cdot d_k$. Hence, $C_k \leq (2 - \frac{1}{m})d_k/(2 - \frac{1}{m}) = d_k$ and \mathcal{S}_{npr} is a feasible schedule.

Moreover, when dividing the execution time of all jobs by $(2 - \frac{1}{m})$, at the same time the speed of each job is multiplied by the same factor. Thus, using Proposition 1 we have that

$$E(\mathcal{S}_{npr}) \leq (2 - \frac{1}{m})^{\alpha-1} E(\mathcal{S}_{pr}) \leq (2 - \frac{1}{m})^{\alpha-1} E(\mathcal{S}^*)$$

since the energy consumed by the optimal preemptive schedule \mathcal{S}_{pr} is a lower bound to the energy consumed by an optimal non-preemptive schedule \mathcal{S}^* for the input instance \mathcal{I} . \blacksquare

Next, we describe how to transform ALGORITHM CRD for $S|r_j = 0, d_j|E$ into ALGORITHM CD for $S|r_j, d_j = d|E$. In order to do this, we modify the Line 3 of ALGORITHM CRD such that to use the Latest Release Date First (LRDF) policy, scheduling the jobs in a backward way starting from the deadline d . Using a similar analysis, the following theorem holds.

Theorem 4 ALGORITHM CD is a $(2 - \frac{1}{m})^{\alpha-1}$ -approximation algorithm for $S|r_j, d_j = d|E$.

4 Clique Instances

In this section, we present a constant factor approximation algorithm for $S|clique|E$. Recall that this problem is \mathcal{NP} -hard as a generalization of $S|r_j = 0, d_j = d|E$. Moreover, in [5] it is proved that $S1|pure-laminar|E$ is \mathcal{NP} -hard. Hence, even $S1|clique|E$ is \mathcal{NP} -hard.

ALGORITHM CL takes as input a clique instance \mathcal{I} and creates first an optimal preemptive schedule, using again one of the algorithms in [2, 4, 8]. Taking into account the execution times of jobs before and after the time $T = \min\{d_j, J_j \in \mathcal{J}\}$ in the preemptive schedule, the algorithm splits the set of jobs into two subsets: the left \mathbb{L} and the right \mathbb{R} . The set \mathbb{L} contains the jobs which have bigger execution time before T , while the set \mathbb{R} consists of the jobs of bigger execution time after T . For the jobs in \mathbb{L} , we modify their deadlines to T and we use ALGORITHM CD. For the jobs in \mathbb{R} , we modify their release dates to T and we use ALGORITHM CRD. Then, ALGORITHM CL returns the concatenation of the two schedules created by ALGORITHM CD and ALGORITHM CRD.

Theorem 5 ALGORITHM CL is a $(2(2 - \frac{1}{m}))^{\alpha-1}$ -approximation algorithm for $S|clique|E$.

Proof. Let $\mathcal{S}_{pr}^{\mathbb{L}}$ and $\mathcal{S}_{pr}^{\mathbb{R}}$ be the preemptive schedules created in Line 1 of ALGORITHM CD and ALGORITHM CRD, respectively. By Theorems 4 and 3, respectively, it holds that

$$E(\mathcal{S}_{npr}) = E(\mathcal{S}_{npr}^{\mathbb{L}}) + E(\mathcal{S}_{npr}^{\mathbb{R}}) \leq (2 - \frac{1}{m})^{\alpha-1} (E(\mathcal{S}_{pr}^{\mathbb{L}}) + E(\mathcal{S}_{pr}^{\mathbb{R}})) \quad (1)$$

Consider, now, the preemptive schedule \mathcal{S} that occurs from \mathcal{S}_{pr} if we schedule the whole work of each job $J_j \in \mathbb{L}$ (resp. $J_j \in \mathbb{R}$) before (resp. after) the time T at the intervals in which J_j is already executed in \mathcal{S}_{pr} . By construction for each job $J_j \in \mathbb{L}$ (resp. \mathbb{R}) it holds that $e_{\ell j} \geq e_{rj}$

ALGORITHM CL(\mathcal{I})

- 1: Create an optimal preemptive schedule \mathcal{S}_{pr} for \mathcal{I} ;
 - 2: Let $e_{\ell j}$ and e_{rj} be the total execution time of each job $J_j \in \mathcal{J}$, before and after, respectively, the time $T = \min\{d_j, J_j \in \mathcal{J}\}$ in \mathcal{S}_{pr} ;
 - 3: Let $\mathbb{L} = \{J_j : e_{\ell j} \geq e_{rj}\}$ and $\mathbb{R} = \{J_j : e_{\ell j} < e_{rj}\}$;
 - 4: Consider the instance $\mathcal{I}_{\mathbb{L}}$ consisting of the jobs in \mathbb{L} , and each $J_j \in \mathbb{L}$ is characterized by a work $w_j(\mathcal{I}_{\mathbb{L}}) = w_j(\mathcal{I})$, a release date $r_j(\mathcal{I}_{\mathbb{L}}) = r_j(\mathcal{I})$, and a deadline $d_j(\mathcal{I}_{\mathbb{L}}) = T$;
 - 5: Run **ALGORITHM CD**($\mathcal{I}_{\mathbb{L}}$) obtaining the non-preemptive schedule $\mathcal{S}_{npr}^{\mathbb{L}}$;
 - 6: Consider the instance $\mathcal{I}_{\mathbb{R}}$ consisting of the jobs in \mathbb{R} , and each $J_j \in \mathbb{R}$ is characterized by a work $w_j(\mathcal{I}_{\mathbb{R}}) = w_j(\mathcal{I})$, a release date $r_j(\mathcal{I}_{\mathbb{R}}) = T$, and a deadline $d_j(\mathcal{I}_{\mathbb{R}}) = d_j(\mathcal{I})$;
 - 7: Run **ALGORITHM CRD**($\mathcal{I}_{\mathbb{R}}$) obtaining the non-preemptive schedule $\mathcal{S}_{npr}^{\mathbb{R}}$;
 - 8: Create the schedule \mathcal{S}_{npr} by concatenating the schedules $\mathcal{S}_{npr}^{\mathbb{L}}$ and $\mathcal{S}_{npr}^{\mathbb{R}}$;
 - 9: **return** \mathcal{S}_{npr} ;
-

(resp. $e_{\ell j} < e_{rj}$). Hence, in order \mathcal{S} to be feasible, we have just to double the speed of each job $J_j \in \mathcal{J}$, and thus by Proposition 1 we have $E(\mathcal{S}, J_j) = 2^{\alpha-1}E(\mathcal{S}_{pr}, J_j)$. In total we get that $E(\mathcal{S}) = 2^{\alpha-1}E(\mathcal{S}_{pr})$.

Let $\mathcal{S}^{\mathbb{L}}$ and $\mathcal{S}^{\mathbb{R}}$ be the parts of \mathcal{S} before and after T , respectively, that is $E(\mathcal{S}) = E(\mathcal{S}^{\mathbb{L}}) + E(\mathcal{S}^{\mathbb{R}})$. The schedules $\mathcal{S}^{\mathbb{L}}$ and $\mathcal{S}_{pr}^{\mathbb{L}}$ concern the same instance, and since $\mathcal{S}_{pr}^{\mathbb{L}}$ is the optimal one, it holds that $E(\mathcal{S}_{pr}^{\mathbb{L}}) \leq E(\mathcal{S}^{\mathbb{L}})$. Similarly, it holds that $E(\mathcal{S}_{pr}^{\mathbb{R}}) \leq E(\mathcal{S}^{\mathbb{R}})$. Therefore, we get

$$E(\mathcal{S}_{pr}^{\mathbb{L}}) + E(\mathcal{S}_{pr}^{\mathbb{R}}) \leq E(\mathcal{S}^{\mathbb{L}}) + E(\mathcal{S}^{\mathbb{R}}) = E(\mathcal{S}) = 2^{\alpha-1}E(\mathcal{S}_{pr}) \quad (2)$$

By combining Equations (1) and (2) and taking into account that $E(\mathcal{S}_{pr})$ is a lower bound to the energy consumed by an optimal non-preemptive schedule \mathcal{S}^* for \mathcal{I} , i.e., $E(\mathcal{S}_{pr}) \leq E(\mathcal{S}^*)$, the theorem follows. \blacksquare

Corollary 1 **ALGORITHM CL** is a $(2(2 - \frac{1}{m}))^{\alpha-1}$ -approximation algorithm for $S|pure-laminar|E$.

5 Agreeable Instances

In this section we deal with agreeable instances and for $\alpha > m$ we improve the known $\alpha^{\alpha}2^{4\alpha}$ -approximation algorithm presented in [3] for $S|agreeable|E$, by proposing a $(4(2 - \frac{1}{m}))^{\alpha-1}$ -approximation algorithm. Recall that, $S|agreeable|E$ is known to be \mathcal{NP} -hard as a generalization of $S|r_j = 0, d_j = d|E$. On the other hand, $S1|agreeable|E$ can be solved in polynomial time [5, 14]. In order to handle an agreeable instance \mathcal{I} , we will first describe how to partition it into clique subinstances, by splitting the set of jobs \mathcal{J} into subsets.

To begin with this partition, we define T_1 to be the smallest deadline of any job in \mathcal{J} , i.e., $T_1 = \min\{d_j : J_j \in \mathcal{J}\}$. Let $\mathcal{J}_1 \subseteq \mathcal{J}$ be the set of jobs which are released before T_1 , i.e., $\mathcal{J}_1 = \{J_j \in \mathcal{J} : r_j \leq T_1\}$. Next, we set $T_2 = \min\{d_j : J_j \in \mathcal{J} \setminus \mathcal{J}_1\}$ and $\mathcal{J}_2 = \{J_j \in \mathcal{J} : T_1 < r_j \leq T_2\}$, and we continue this process until all jobs are assigned into a subset of jobs (see Figure 2). Let k be the number of subsets of jobs that have been created.

In the above partition, the subsets of jobs are pairwise disjoint. However, the two clique subinstances induced by two subsets of jobs may have time interference. In other words, there may exist two jobs $J_j \in \mathcal{J}_{\ell}$ and $J_{j'} \in \mathcal{J}_{\ell'}$, where $0 \leq \ell < \ell' \leq k$, such that $d_j \geq r_{j'}$. Nevertheless, the key observation here is that in the above partition into clique subinstances only two consecutive

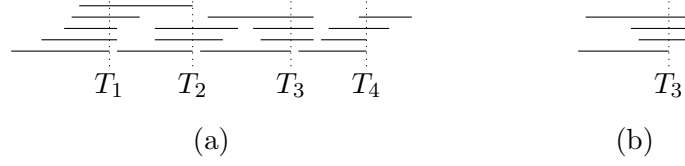


Figure 2: (a) The partition of jobs. (b) The subset of jobs \mathcal{J}_3 .

subsets of jobs may have time interference. In fact a stronger property holds: for each ℓ , $1 \leq \ell \leq k-1$, and each job $J_j \in \mathcal{J}_\ell$, the active interval of J_j does not contain $T_{\ell+1}$. To see this, given a partition into the subsets of jobs $\mathcal{J}_1, \mathcal{J}_2, \dots, \mathcal{J}_k$ and the corresponding times T_1, T_2, \dots, T_k , assume for contradiction that there is a job $J_j \in \mathcal{J}_\ell$ whose active interval contains $T_{\ell+1}$. Hence, it holds that $r_j \leq T_\ell$ and $d_j > T_{\ell+1}$. Consider, now, the job $J_{j'} \in \mathcal{J}_{\ell+1}$ whose deadline defines $T_{\ell+1}$. For $J_{j'}$ it holds that $r_{j'} > T_\ell$ and $d_{j'} = T_{\ell+1}$. Thus, we have that $r_j < r_{j'}$ and $d_j > d_{j'}$, which is a contradiction that our instance is agreeable.

Next, we propose ALGORITHM AGR which initially defines the times T_1, T_2, \dots, T_k and the corresponding subsets of jobs $\mathcal{J}_1, \mathcal{J}_2, \dots, \mathcal{J}_k$. Then, in order to separate the intervals of two consecutive subsets of jobs, our algorithm decreases the active intervals of all jobs by half. This will ensure the feasibility of the schedule for the whole instance \mathcal{I} . For each clique subinstance, ALGORITHM CL is called and it gives a non-preemptive schedule. Our algorithm returns the concatenation of the schedules for the clique subinstances.

ALGORITHM AGR

- 1: Find the partition into the subsets of jobs $\mathcal{J}_1, \mathcal{J}_2, \dots, \mathcal{J}_k$ and the corresponding times T_1, T_2, \dots, T_k ;
 - 2: **for** $\ell = 1$ to k **do**
 - 3: Consider the clique instance \mathcal{I}_ℓ consisting of the jobs in \mathcal{J}_ℓ , and each $J_j \in \mathcal{J}_\ell$ is characterized by a work $w_j(\mathcal{I}_\ell) = w_j(\mathcal{I})$, a release date $r_j(\mathcal{I}_\ell) = r_j(\mathcal{I}) + \frac{T_\ell - r_j(\mathcal{I})}{2}$, and a deadline $d_j(\mathcal{I}_\ell) = d_j(\mathcal{I}) - \frac{d_j(\mathcal{I}) - T_\ell}{2}$;
 - 4: Run ALGORITHM CL(\mathcal{I}_ℓ) obtaining the non-preemptive schedule $\mathcal{S}_{npr}^{(\ell)}$;
 - 5: Create the schedule \mathcal{S}_{npr} by concatenating the schedules $\mathcal{S}_{npr}^{(1)}, \mathcal{S}_{npr}^{(2)}, \dots, \mathcal{S}_{npr}^{(k)}$;
 - 6: **return** \mathcal{S}_{npr} ;
-

The following proposition deals with two clique instances consisting of the same set of jobs with different active intervals (as in Line 3 of ALGORITHM AGR). Note that the proposition holds both for preemptive and non-preemptive schedules.

Proposition 6 *Consider a clique instance \mathcal{I}_1 consisting of a set of jobs \mathcal{J}_1 and let $T = \min\{d_j : J_j \in \mathcal{J}_1\}$. Let \mathcal{I}_2 be an instance consisting of the set of jobs $\mathcal{J}_2 = \mathcal{J}_1$ and for each $J_j \in \mathcal{J}_2$ we have that $w_j(\mathcal{I}_2) = w_j(\mathcal{I}_1)$, $r_j(\mathcal{I}_2) = r_j(\mathcal{I}_1) + \frac{T - r_j(\mathcal{I}_1)}{2}$ and $d_j(\mathcal{I}_2) = d_j(\mathcal{I}_1) - \frac{d_j(\mathcal{I}_1) - T}{2}$. For two optimal schedules \mathcal{S}_1 and \mathcal{S}_2 for \mathcal{I}_1 and \mathcal{I}_2 , respectively, it holds that $E(\mathcal{S}_2) \leq 2^{\alpha-1} E(\mathcal{S}_1)$.*

Proof. Note first that the active interval of each job $J_j \in \mathcal{J}_1$ from $d_j(\mathcal{I}_1) - r_j(\mathcal{I}_1)$ in \mathcal{I}_1 becomes $d_j(\mathcal{I}_2) - r_j(\mathcal{I}_2) = \frac{1}{2}(d_j(\mathcal{I}_1) - r_j(\mathcal{I}_1))$ in \mathcal{I}_2 .

Given an optimal schedule \mathcal{S}_1 for \mathcal{I}_1 , we can obtain a feasible schedule \mathcal{S}'_2 for \mathcal{I}_2 if we decrease the execution time of each job $J_j \in \mathcal{J}_1$ by a factor of $\frac{1}{2}$ and then compact the schedule keeping

fixed the time T . In order to do this, we have to increase the speed of J_j by a factor of 2. Hence, by Proposition 1 it holds that $E(\mathcal{S}'_2) = 2^{\alpha-1}E(\mathcal{S}_1)$. As \mathcal{S}_2 is the optimal schedule for \mathcal{I}_2 , the proposition follows. ■

Theorem 7 ALGORITHM AGR is a $(4(2 - \frac{1}{m}))^{\alpha-1}$ -approximation algorithm for $S|agreeable|E$.

Proof. We first deal with the approximation ratio of the algorithm. Let $\mathcal{S}_{pr}^{(1)}, \mathcal{S}_{pr}^{(2)}, \dots, \mathcal{S}_{pr}^{(k)}$ be the optimal preemptive schedules created in Line 1 of ALGORITHM CL with input the instances $\mathcal{I}_1, \mathcal{I}_2, \dots, \mathcal{I}_k$, respectively. By Theorem 5, for each ℓ , $1 \leq \ell \leq k$, it holds that $E(\mathcal{S}_{npr}^{(\ell)}) \leq (2(2 - \frac{1}{m}))^{\alpha-1}E(\mathcal{S}_{pr}^{(\ell)})$, and hence

$$E(\mathcal{S}_{npr}) = \sum_{\ell=1}^k E(\mathcal{S}_{npr}^{(\ell)}) \leq (2(2 - \frac{1}{m}))^{\alpha-1} \sum_{\ell=1}^k E(\mathcal{S}_{pr}^{(\ell)}) \quad (3)$$

For each ℓ , $1 \leq \ell \leq k$, let $\mathcal{S}^{(\ell)}$ be the optimal preemptive schedule for the jobs in \mathcal{J}_ℓ subject to their original release dates and deadlines in \mathcal{I} . By Proposition 6 we have that $E(\mathcal{S}_{pr}^{(\ell)}) \leq 2^{\alpha-1}E(\mathcal{S}^{(\ell)})$, and hence

$$\sum_{\ell=1}^k E(\mathcal{S}_{pr}^{(\ell)}) \leq 2^{\alpha-1} \sum_{\ell=1}^k E(\mathcal{S}^{(\ell)}) \quad (4)$$

Let \mathcal{S}_{pr} be an optimal preemptive schedule for the whole instance \mathcal{I} . Note that, $\sum_{\ell=1}^k E(\mathcal{S}^{(\ell)})$ is a lower bound to $E(\mathcal{S}_{pr})$, since all jobs in $\bigcup_{\ell=1}^k \mathcal{S}^{(\ell)}$ have their original release dates and deadlines, and each of $\mathcal{S}^{(\ell)}$ is an optimal preemptive schedule. Hence, by combining the Equations (3) and (4), the approximation ratio achieved by our algorithm follows.

In order to show the feasibility of the schedule \mathcal{S}_{npr} created by ALGORITHM AGR, it suffices to show that for each ℓ , $1 \leq \ell \leq k-1$, and for any two jobs $J_j \in \mathcal{J}_\ell$ and $J_{j'} \in \mathcal{J}_{\ell+1}$ it holds that $d_j(\mathcal{I}_\ell) < r_{j'}(\mathcal{I}_{\ell+1})$. More intuitively, we want to show that the active intervals of any two jobs belonging to two consecutive (w.r.t. time) instances does not intersect.

By construction, it holds that $d_j(\mathcal{I}) \leq T_{\ell+1}$ and $r_{j'}(\mathcal{I}) > T_\ell$. Moreover, we have that $d_j(\mathcal{I}_\ell) = d_j(\mathcal{I}) - \frac{d_j(\mathcal{I}) - T_\ell}{2} \leq \frac{T_{\ell+1} + T_\ell}{2}$ and $r_{j'}(\mathcal{I}_{\ell+1}) = r_{j'}(\mathcal{I}) + \frac{T_{\ell+1} - r_{j'}(\mathcal{I})}{2} > \frac{T_{\ell+1} + T_\ell}{2}$. Therefore, $d_j(\mathcal{I}_\ell) < r_{j'}(\mathcal{I}_{\ell+1})$, and the theorem follows. ■

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